MÄLARDALEN UNIVERSITY
DEPARTMENT OF MATHEMATICS AND PHYSICS
ANALYTICAL FINANCE II – MT1470
RESPONSIBLE TEACHER: JAN RÖMAN

COURSE SEMINAR

DATE: 10 DECEMBER 2004

Caps, Floors and Collars

GROUP 4:

DANIELA DE OLIVEIRA ANDERSSON
HERVÉ FANDOM TCHOMGOUO
XIN MAI
LEI ZHANG

Abstract

In the course Analytical Finance II, we continue discuss the common financial instruments in OTC markets. As an extended course to Analytical Finance I, we go deep inside the world of interest rate and its derivatives. For exploring the dominance power of interest rate, some modern models were derived during the course hours and the complexity was shown to us, exclusively.

The topic of the paper is about interest rate Caps, Floors and Collars. As their counterparts in the equity market, as Call, Put options and strategies. We will try to approach the definitions from basic numerical examples, as what we will see later on.

Table of Contents

1. Introduction

From course in corporate finance, we know the importance of interest rate to modern corporations. Due to the frequent cash flows in these corporations are based on financial papers, the effect of interest rate can be hardly ignored. In the equity market, in order to hedge the risk from underlying stocks, we create stock options as means of managing investment risk. In the interest rate market, we find the same financial products, called interest rate Caps, Floors, and Collars.

Interest rate Caps and Floors are basic products in hedging floating rate risk. They set the minimum return levels on one side of interest rate movement and allow the profit on the other side. Caps and Floors are counterparts to Call and Put options in equity market. In more detail, they are composition of individual options, called Caplets and Floorlets. By the help of these interest rate derivatives, corporations enjoy much freedom in managing financial assets and liabilities. In conjunction with other financial instruments, Caps and Floors may help remapping the corporation risk structure.

The paper will take Caps, Floors and their composition Collars as equally weighted individuals. They will be introduced in the same format as starting from basic concepts, numerical example, mathematical modeling, and theoretical example. As commonly accepted that floors are inversion of caps, we put more detailed introduction on caps. After the comparison explanation on floors, we continue to collars. Basically, interest rate collars are strategic composition of caps and floors. As proceeding to collars, we turn to concern more on the practical uses these interest rate derivatives, strategies.

Also, as a summarizing of the course Analytical Finance II, by collecting all the information after the course, we focus on the practical calculations and presence of these key derivatives. We avoid the deep proof on the mathematical models for the reasons of repetition or complexity. We strongly believe that base on the understanding of the usage in this paper; the reader may find the linkage between the market presence and the mathematical models we met during the course. Also, as a presentation after this course, this paper is a collection and interpretation on Internet materials. Based on the materials, we rewrite the concepts on our own understanding.

As other students in the same course, we gradually realize the importance of financial derivates and their real meaning to financial practitioners and analysts. As a basic training for future career, we will more concentrate on financial derivatives and their flexible usage.

2. Caps and caplets

2.1 Concept

A cap is a call option on the future realization of a given underlying LIBOR rate. More specifically, it is a collection of caplets, each of which is a call option on the LIBOR at a specified date in the future. A series of caplets or cap can extend for up to 10 years in most markets.

Banks and institutions will use caps to limit their risk exposure to upward movements in short term floating rate debt. Caps are equally attractive to speculators as considerable profits can be achieved on volatility plays in uncertain interest rate environments.

In the case of institutions where the maturities and duration of their assets, such as mortgages, exceed the maturities of their liabilities, usually short term deposits are vulnerable to rises in short term interest rates as the cost of funding will rise without any comparable increase in earnings from its mortgage assets. The institution may wish to convert the floating rate debt to fixed rate payments using an interest rate swap. However weak credit rating may limit or preclude access to the interest rate swap market for some of these institutions. Caps can be an effective alternative in this scenario, as the risk of a rate rise can be hedged without any consideration of credit risk for the writer once the premium has been received.

Whilst uncommon in Japan caps can have applications in Leveraged Buyouts (LBO's). Institutions involved in any LBO will invariably take on a considerable amount of short term floating rate debt whether it is to fund strategies to defend against, or finance a hostile takeover bid. A successful bid will increase the institutions debt to equity ratio significantly and even a small rise in interest rates could be disastrous. The credit rating of the institution at this stage is unlikely to permit access to the IRS market so the purchase of 2-year or 3-year caps would hedge the interest rate risk during the period of reorganization following the LBO.

Volatility provides speculators with an essential benchmark for trading in caps and other interest rate options where most structures are highly customized and the monitoring of risk is extremely complicated. This financial instrument is primarily used by borrowers of floating rate debt in situations where short term interest rates are expected to increase. For highly leveraged companies or those with an overweighting of short term debt, rate caps are used to manage interest expense and therefore, the profitability profile of the organization. Rate caps can thus be viewed as insurance, ensuring that the maximum borrowing rate never exceeds the specified cap level. In exchange for this peace of mind, the purchaser pays the financial institution a premium. Caps generally guarantee a maximum level of either 3- or 6-month LIBOR or whatever the prevailing floating rate index is in the particular market. The clients' maximum loss on a cap transaction is the premium.

In the Yen market caps usually have maturities of 1 to 10 years and are based on LIBOR. The standard reset frequency is 3 months for 1 year and 6 months for 2 years out to 10 years, other reset frequencies are possible, e.g. 1 year against 1 month, but uncommon. The standard amount is Y10 billion.

As usual with OTC options all parameters of the cap are negotiable but the bid-offer spread will widen as the cap becomes more complicated and therefore the transaction cost may increase substantially. Many participants in the market will absorb any mismatch risk between their position and a more standardized cap structure to take advantage of the cheaper cost and greater liquidity.

2.2 Pricing Interest Rate Cap

A Cap is a series of sequentially maturing European style call options that protect the purchaser from a rise in a floating rate index, usually LIBOR, above a predetermined level. The purchaser has the right to receive a periodical cash flow equal to the difference between the market rate and the strike, effectively placing a maximum limit on interest payments on floating rate debt.

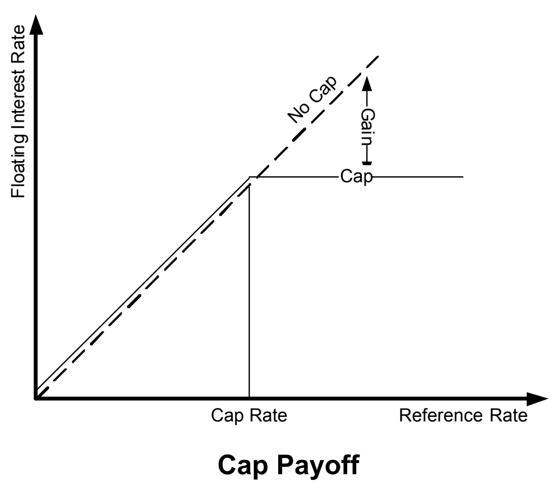
Consider a cap with a total life of T, a principal of L, and a cap rate of R_K. Suppose that the reset dates are t_1, $t_2 \ldots t_n$ and define $t_{n+1} = T$. Define R_K as the interest rate for the period between time t_k and t_{k+1} observed at time t_k ($1 \le k \le n$). The cap leads to a payoff at time t_{k+1} ($k = 1, 2\ldots n$) of

$$Ld_k \max(R_k - R_K, 0) \qquad (2.2.1)$$

where $d_k = t_{k+1} - t_k$. (Both R_k and R_K are expressed with compounding frequency equal to the frequency of resets.)

The equation (2.2.1) is a call option on the LIBOR rate observed at time t_k with the payoff occurring at time t_{k+1}. The cap portfolio of n such options. LIBOR rates are observed at times t_1, $t_2 \ldots t_n$ and the corresponding payoffs are occurred at times t_2, $t_3 \ldots t_{n+1}$. The n call options underlying are known as caplets. The payoff diagram is shown as in Figure 2.1.1.

Figure 2.1.1 Interest rate cap



An interest rate cap can also be characterized as a portfolio of put options on zero-coupon bonds with payoffs on the puts occurring at the time they are calculated. The payoff at time t_{k+1} is equivalent to

$$\frac{Ld_k}{1 + R_k d_k} \max(R_k - R_K, 0) \text{ or } \max\left(L - \frac{L(1 + R_K d_k)}{1 + R_k d_k}, 0\right) \qquad (2.2.2)$$

The expression $\dfrac{L(1+R_k d_k)}{1+R_k d_k}$ is the value of a zero-coupon bond that pays off $L(1+R_k d_k)$ at time t_{k+1}. The expression in equation (2.2.2) is therefore the payoff from a put option, with maturity t_k, on a zero-coupon bond with maturity t_{k+1} when the face value of the bond is $L(1+R_k d_k)$ and the strike price is L. It follows that an interest rate cap can be regarded as a portfolio of European put options on zero-coupon bonds.

2.3 Numerical Example[1]

Consider an institution that has issued a floating rate note for which the interest rate is reset every 90 days, using the prevailing 90-day spot interest rate. The next reset day is in 42 days' time. The institution has purchased an interest rate cap with maturity of 42 days and cap rate of 5.50 percent.

In 42 days, at maturity, the payoff to the cap is defined by

$$\text{Cap }(42) \equiv \max\left\{\frac{i_s(90)-0.055}{1+i_s(90)\times(90/360)},0\right\}\times\frac{90}{360}\times\text{Principle} \tag{2.3.1}$$

where $i_s(90)$ is the 90-day spot interest rate observed in 42 days, and *Principal* is the notional principal amount of the floating rate note. The cap payoff is discounted because the payoff to the cap is received 90 more days further into the future than the maturity of the cap. In this example, a 360-day year convention is used in the definition of the spot interest rate, and simple interest rate are used.

For example, if the 90-day spot interest rate at 42 days is 5.80 percent, a principal of \$10 million, then the payoff to the cap is

$$\begin{aligned}\text{Cap }(42) &\equiv \max\left\{\frac{0.058-0.055}{1+0.058\times(90/360)},0\right\}\times\frac{90}{360}\times 10{,}000{,}000\\ &\equiv \max\{0.002957,0\}\times\frac{90}{360}\times 10{,}000{,}000\\ &= \$7{,}392.80\end{aligned}$$

The institution can take the money at the maturity of the cap and invest it for 90 days at the 90-day spot interest. Therefore, in 90 days the institution pays the interest on its floating rate note and receives the money from its cap investment. The net payment is

$$\begin{aligned}&= 10{,}000{,}000\times 0.058\times\frac{90}{360}-7{,}392.80\times\left[1+0.058\times\frac{90}{360}\right]\\ &= \$137{,}550\end{aligned}$$

implying that the effective interest rate the institution pays in the cap rate of 5.50 percent; for an interest rate of 5.50 percent, the payment is $10{,}000{,}000\times 0.055\times\dfrac{90}{360}=\$137{,}550$.

[1] See Jarrow 528.

2.4 Pricing Interest Rate Cap – Black-Scholes

The cap is a series of European style call options, caplets, and its price is the sum of these caplets. The dominant pricing method in use is the Black-Scholes option pricing methodology.

The caplet corresponding to the rate observed at time t_k provides a payoff at time t_{k+1} of $L\delta_k \max(R_k - R_K, 0)$.

If the rate R_K is assumed to be lognormal with volatility σ_k, equation (2.3.1) gives us the value of this caplet as

$$L\delta_k P(0, t_{k+1})\left[F_k N(d_1) - R_k N(d_2)\right]$$

$$d_1 = \frac{\ln(F_k / R_k) + \sigma_k^2 t_k / 2}{\sigma_k \sqrt{t_k}} \qquad (2.3.1)$$

$$d_2 = \frac{\ln(F_k / R_k) - \sigma_k^2 t_k / 2}{\sigma_k \sqrt{t_k}} = d_1 - \sigma_k \sqrt{t_k}$$

where F_k is the forward rate for the period between time t_k and t_{k+1}.

2.5 Theoretical Example [2]

Consider a contract that caps the interest rate on a $10,000 loan at 8% per annum (with quarterly compounding) for three months starting in one year. This is a caplet and could be one element of a cap. Suppose that the zero-curve is flat at 7% per annum with quarterly compounding and the one-year volatility for the three-month rate underlying the caplet is 20% per annum. The continuously compounded zero rate for all maturities is 6.9394%. $F_k = 0.07$, $\delta_k = 0.25$, $L = 10{,}000$, $R_K = 0.08$, $t_k = 1.0$, $t_{k+1} = 1.25$, $P(0, t_{k+1}) = e^{-0.069394 \times 1.25} = 0.9169$, and $\sigma_k = 0.20$. Also,

$$d_1 = \frac{\ln(0.07/0.08) + 0.2^2 \times 1/2}{0.20 \times 1} = -0.5677$$

$$d_2 = d_1 - 0.20 = -0.7677$$

so that the caplet price is $0.25 \times 10{,}000 \times 0.9169\left[0.07 N(-0.5677) - 0.08 N(-0.7677)\right] = \$5.162$

Each caplet of a cap must be valued separately. One approach is to use a different volatility for each caplet. The volatilities are then referred to as spot volatilities. An alternative approach is to use the same volatility for all caplets constituting any particular cap but to vary this volatility according to the life of the cap. The volatilities used are then referred as flat volatilities. The volatilities quoted in the market are usually flat volatilities. Thus, the simplicity of the Black-Scholes model is tempered by its assumption that short term interest rates are constant. Options on short term interest rate will only have value if rates are

[2] See Hull 518.

stochastic, i.e. not completely predictable. Therefore many other stochastic interest rate models have been developed by financial economists to compete with the Black-Scholes model as the definitive derivative pricing model for market participants.

2.6 Advantage and Disadvantage

Major advantages of caps are that the buyer limits his potential loss to the premium paid, but retains the right to benefit from favorable rate movements. The borrower buying a cap limits exposure to rising interest rates, while retaining the potential to benefit from falling rates. An upper limit is therefore placed on borrowing costs.

Caps are highly flexible and versatile in that notional amounts, strike rates and expiry dates can be structured in a way that can provide tailor-made solutions for clients. Caps are also highly suited for hedging contingent future commitments.

Major advantages of caps are that the buyer limits his potential loss to the premium paid, but retains the right to benefit from favorable rate movements.

Interest rate option products are highly geared instruments and, for a relatively small outlay of capital, purchasers can make considerable profits. At the same time, a seller with a decay strategy in mind (i.e. where he would like the option's value to decay over time so that it can be bought back cheaper at a later stage or even expire worthless), can make a profit amounting to the option premium, without having to make a capital outlay.

The disadvantages of caps are that the premium is a non-refundable cost which is paid upfront by the buyer, and the negative impact of an immediate cash outflow. Caps can theoretically lose all their value (i.e. the premium paid) if they expire as out-the-money or start to approach their expiry dates. In addition, there are high potential losses for writers (sellers) of option-type interest rate derivatives if market movements are contrary to market expectations. Also, it needs to keep in mind that the bid/offer spreads on most option-type interest rate derivative products are quite wide.

2.7 Strategies[3]

The Cap is a guarantee of a future rate. The implied forward rate will change over time as the market changes its view of future rates. The price of the Cap will therefore depend on the likelihood that the market will change its view. This likelihood of change is measured by volatility. An instrument expected to be volatile between entry and maturity will have a higher price than a low volatility instrument. The volatility used in calculating the price should be the expected future volatility. This is based on the historic volatility.

As time goes by, the volatility will have less and less impact on the price, as there is less time for the market to change its view. Therefore, in a stable market, the passing of time will lead to the Cap FALLING in value. This phenomenon is known as Time Decay. This increases in severity as we get closer to maturity. The higher the strike compared to prevailing interest rates the lower the price of the cap. High strike ("*out of the money*") caps will be cheaper than "*at the money*" or low strike ("*in the money*") because of the reduced probability of the caplets being in the money during the life of the option. The price of the cap will increase

[3] See CiberConta.

with the length of the tenor as it will include more caplets to maturity. The further the strike is set *out of the money*, the cheaper the Cap, as the probability of payout is less, therefore the Cap is considered to be more LEVERAGED. As rates rise the Cap will increase in value as it becomes closer to the money.

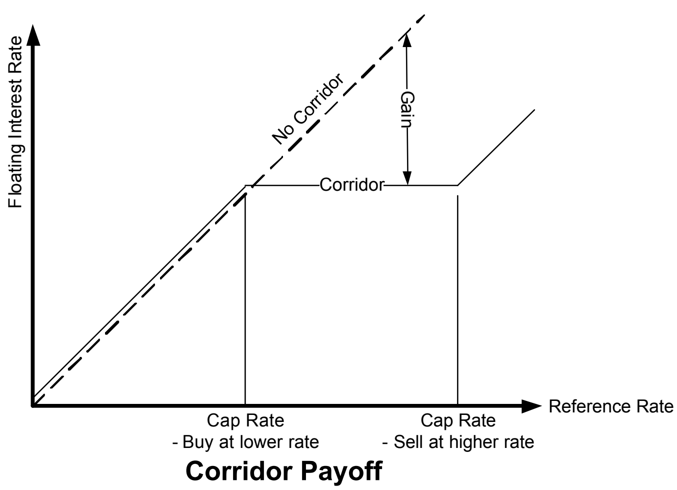
It is therefore an interesting strategy to buy OUT OF THE MONEY Caps for a small premium which will increase in value dramatically (due to the leverage) as rates rise. The Cap can then be sold. This is a trading strategy rather than buys and holds strategy. Sophisticated Investors or Borrowers may like to SELL Caps. This is also known as writing Caps. In this case the seller is PROVIDING the guarantee and therefore has an unlimited loss potential. The profit from this strategy is limited to the premium earned and will occur when there are no claims against the Cap.

In the market traders will use volatility to quantify the probability of changes around interest rate trends. Higher volatility will increase the probability of a caplet being in the money and therefore the price of the cap.

❖ Corridor

It is a strategy where the cost of purchasing a cap is offset by the simultaneous sale of another cap with a higher strike. It is possible to offset the entire cost of the cap purchase by increasing the notional amount on the cap sold to match the purchase price. The inherent risk in this strategy is that if short term rates rise through the higher strike the purchaser is no longer protected above this level and will incur considerable risk if the amount of the cap sold is proportionately larger. The payoff diagram is shown as in Figure 2.7.1.

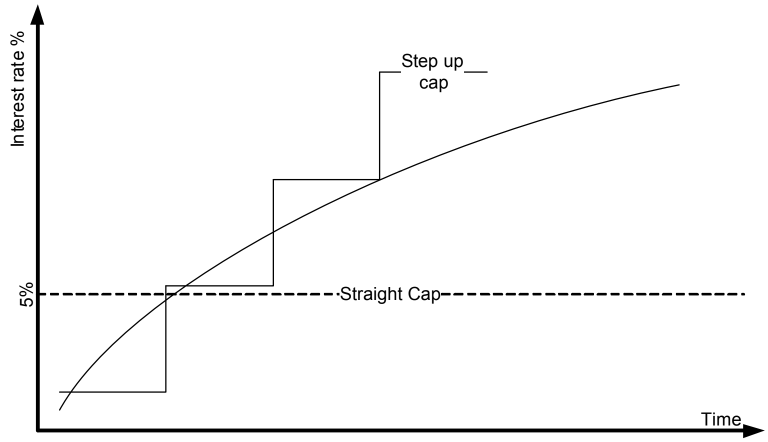
Figure 2.7.1 Corridor Strategy



❖ Step up cap

In steep yield curve environments the implied forward rates will be much higher than spot rates and the strike for caplets later in the tenor may be deep in the money. The price of a cap, being the sum of the caplets, may prove prohibitively expensive. The step up cap counteracts this by raising the strike of the later caplets to reflect the higher forward rates. This may provide a more attractive combination of risk hedge at a lower price. The payoff diagram is shown as in Figure 2.7.2.

Figure 2.7.2 Step-up Cap Strategy



Source: CiberConta.

After purchasing the Cap, the buyer can make "claims" under the guarantee should Libor be above the level agreed on the Cap on the settlement dates. A Cap is NOT a continuous guarantee; claims can only be made on specified settlement dates. These dates are selected by the purchaser.

3. Floors and floorlets

3.1 Concepts

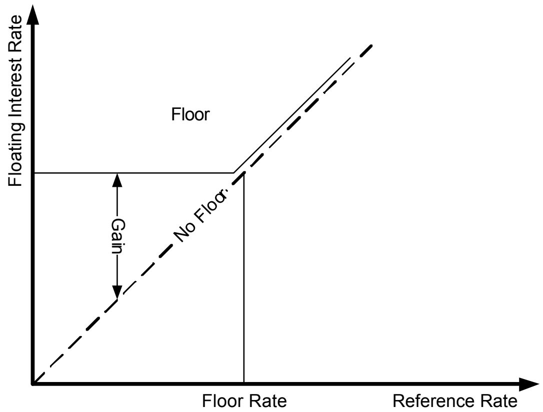
Interest rate floor is similar to cap except that it is structured to hedge against decreasing interest rates (or down-side risk). Interest rate floor can be purchased from a bank at an amount to a contract. As a contract, when a chosen reference rate falling below the floor's interest rate level (the floor rate), the interest floor seller agrees to reimburse the buyer for the difference, calculated on a notional principal amount and for a certain period. Therefore, the chosen reference rate must drop below the floor rate before any cash payment takes place between the two parties.

An interest rate floor closely resembles a portfolio of put option contracts. As option contracts, to take the risk of compensating, the sellers ask the buyers a premium payment. The key elements of a floor are maturity, floor rate, reference floating rate, reset period and the notional principal amount. The payoff diagram of floor is shown in Figure 3.1.1.

A typical interest rate floor can be considered as a composition of many interest rate floorlets, which only yield payment on one period of time. We can consider floorlet as a European put on the chosen reference rate with delayed payment of the payoff.

For an example, a firm has cash of $1,000,000 and thinks to have it for some time. It does not want to negotiate a fixed rate of interest, because interest rates might be considered to rise, but it does want to guarantee a certain minimum return. In such a case, interest floor will be an ideal product for the firm to purchase. It guarantees the firm a minimum rate of return on the cash deposit. If the floating interest rate runs below the minimum rate, the seller/bank will compensate the loss on falling interest rate. On the other hand, the firm still can enjoy a higher interest rate on the concerned rising floating interest rate.

Figure 3.1.1 **Interest rate floor**

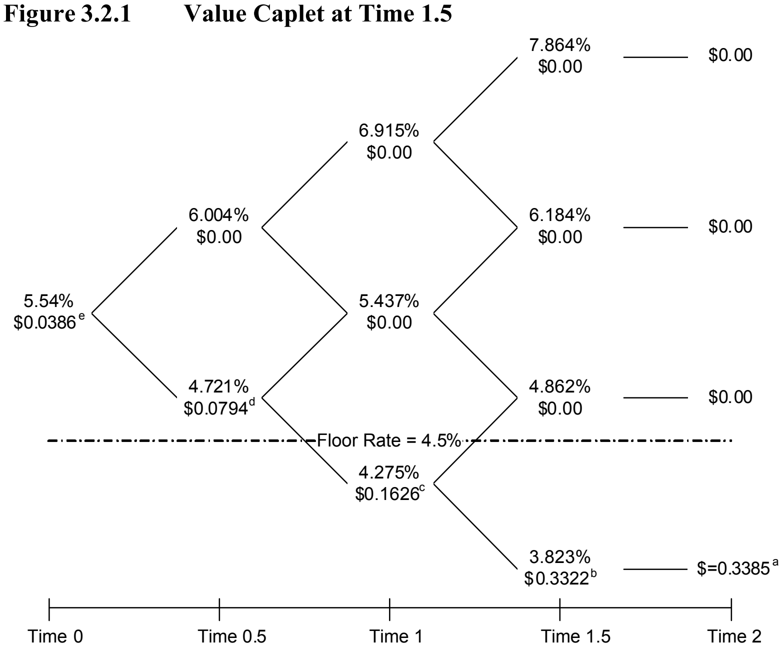


Floor Payoff

3.2 Numerical Example[4]

Consider a 2-year semi-annual floor on $100 notional amount with strike rate k = 4.5%, indexed to the 6-month rate. At time 0, the 6-month rate is 5.54 percent so the floor is out-of-the-money, and pays 0 at time 0.5. The later payments of the floor depend on the path of interest rates. Suppose rates follow the path in the tree below in Figure 2. The value of the floor is the sum of the values of the 4 puts on the 6-month rates at times 0, 0.5, 1, and 1.5. We begin from valuing at Time 1.5

Figure 3.2.1 **Value Caplet at Time 1.5**



As the binomial valuation of put options, we calculate back from Time 1.5 as in Figure 3.2.1. At Time 1.5, the only possible floating interest rate below the floor rate of 4.5 percent is 3.823

[4] See NYU Stern.

percent. And, only when the floating interest rate falling to this level, the buyer of the floor will be compensated with the cash of $33.85 at Time 2. For the calculation,

$$\$0.3385 = \$100 * (4.5\% - 3.823\%) / 2 \quad \text{(a)}$$

For the value of $33.85, at Time 1.5, its present values is,

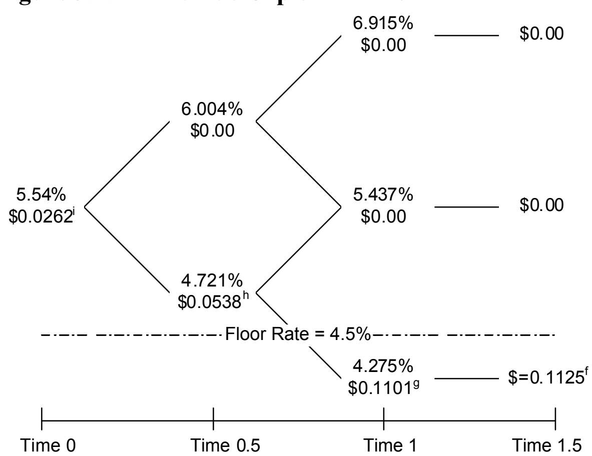
$$\$0.3322 = \$0.3385 / (1 + 3.823\% / 2) \quad \text{(b)}$$

For Time 1, 0.5 and 0, assume that in this example, the probabilities to raise and fall are both 50 percent. The calculations for the values at these time spots will be:

$$\$0.1626 = 0.5 * (0 + \$0.3322) / (1 + 4.275\% / 2) \quad \text{(c)}$$
$$\$0.0794 = 0.5 * (0 + \$0.1626) / (1 + 4.721\% / 2) \quad \text{(d)}$$
$$\$0.0386 = 0.5 * (0 + \$0.0794) / (1 + 5.54\% / 2) \quad \text{(e)}$$

Then, we calculate the floorlet due at Time 1 as in Figure 3.2.2.

Figure 3.2.2 Value Caplet at Time 1

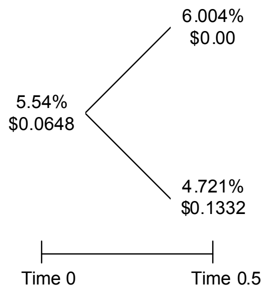


Base on the same calculation method, we simply give the calculation for each node as following:

$$\$0.1125 = \$100 * (4.5\% - 4.275\%) / 2 \quad \text{(f)}$$
$$\$0.1101 = \$0.1125 / (1 + 4.275\% / 2) \quad \text{(g)}$$
$$\$0.0538 = 0.5 * (0 + \$0.1101) / (1 + 4.721\% / 2) \quad \text{(h)}$$
$$\$0.0262 = 0.5 * (0 + \$0.0538) / (1 + 5.54\% / 2) \quad \text{(i)}$$

Because at Time 0.5 and 0, the floorlets never get in the money, so the value of the floor will be $ 0.0648 = $0.0386 + $0.0262.

Figure 3.2.3 Value Caplet at Time 0



3.3 Mathematical pricing[5]

Technically, the payoff of floor at time t_i (i = 1… n) is given by:

$$F_{t_{i-d}}^{i} = Nd \max\left\{K - l\left(t_i, t_{i-d}\right), 0\right\} \tag{3.3.1}$$

where K denotes the interest floor rate, N denotes the notional principle amount and $l(t_i,t_{i-d})$ is the forward rate or the reference rate as mentioned before. As we explained, a floor is a composition of floorlets at different maturity dates. As reviewing equation (3.3.1), the *ith* floorlet of a floor can be expressed in the following way:

$$F_{t_{i-d}}^{i} = N\left(1+dK\right)\max\left\{P_{t_{i-d}}^{t_i} - \frac{1}{1+dK}, 0\right\} \tag{3.3.2}$$

The accumulated value of a floor at any time t < t_0 is given by:

$$F_t = N\left(1+dK\right)\sum_{i=1}^{n} p\left\{t; \frac{1}{1+dK}, t_{i-d}, t_i\right\} \qquad t < t_0 \tag{3.3.3}$$

Further, the value of a floor is given by:

$$F_t = NP_t^{t_{i(t)}} d \max\left\{K - l(t_{i(t)}, t_{i(t)-d}), 0\right\} + N\left(1+dK\right)\sum_{i=i(t)+1}^{n} p\left(t; \frac{1}{1+dK}, t_{i-d}, t_i\right)$$
$$t \le t_0 \le t_n \tag{3.3.4}$$

Because the put option character of floor, if R_i, the interest rate between time t_i and t_{i+1}, is assumed to be lognormal with volatility s_i. The famous Black and Scholes formula may be occupied for calculating the value of a floorlet. By introducing Black and Scholes formula into equation (3.3.5), we get the Black and Scholes formula for floorlet as:[6]

$$F_{t_{i-d}}^{i} = Nd\,P\left(0, t_{i+1}\right)\left[KN\left(-d_2\right) - l\left(t_i, t_{i-d}\right)N\left(-d_1\right)\right] \tag{3.3.5}$$

[5] See Röman 24-25.
[6] See Hull 517-518.

3.4 Theoretical Example[7]

Consider a contract that floors the interest rate on a \$10,000 loan at 8% per annum (with quarterly compounding) for three months starting in one year. This is a floorlet and could be one element of a floor. Suppose that the zero curves is flat at 9% per annum with quarterly compounding and the one-year volatility for the three-month rate underlying that floorlet is 20% per annum. The continuously compounded zero rate for all maturities is 6.9394%. In equation (3.3.5), K = 8%, d = 0.25, N = 10,000, $l(t_i, t_{i-d})$ = 9%, t_i = 1.0, t_{i+1} = 1.25 and s_k = 0.20. Then,

$$P(0, t_{i+1}) = e^{-0.069394*1.25} = 0.9619$$

$$d_1 = \frac{\ln(0.09/0.08) + 0.2^2 \times 0.5}{0.20 \times 1} = 0.6889, \qquad d_2 = d_1 - 0.20 = 0.4889$$

$$F_{t1}^{1.25} = 10{,}000 \times 0.25 \times 0.9169 \left[0.08 \times N(-0.4889) - 0.09 N(-0.6889)\right] = \$6.663$$

4. Collars

4.1 Concepts

Combining a cap and a floor into one product creates a "Collar". Collars can benefit both borrowers and investors. In the case of a borrower, the collar protects against rising rates but limits the benefits of falling rates. In the case of an investor, the collar protects against falling rates but limits the benefits of rising rates. Similar to Caps and Floors the customer selects the index (Prime, LIBOR, C.P., PSA), the length of time, and strike rates for both the cap and the floor. However, unlike a cap or a floor, an up-front premium may or may not be required, depending upon where the strikes are set. In either scenario, the customer is a buyer of one product, and a seller of the other.

4.2 Strategies

The buyer and the seller agree upon the term (tenor), the cap and floor strike rates, the notional amount, the amortization ("bullet", mortgage, straight line, etc.), the start date, and the settlement frequency. If at any time during the tenor of the collar, the index moves above the cap strike rate or below the floor strike rate, one party will owe the other a payment. The payment is calculated as the difference between the strike rate and the index times the notional amount outstanding times the day's basis for the settlement period. (Note: the exact calculation is contingent upon the specific terms and conditions of the contract. C.P. and Prime use a daily average for the settlement periods).

4.3 Numerical Example

A customer is borrowing \$10 million at 1 mo. LIBOR plus 200 bps, for a current rate of 7.75% (LIBOR is currently at 5.75%), from ABC Bank. The customer wishes to cap LIBOR so that it does not exceed 6%. In order to reduce the cost of the cap, the borrower sells a floor to ABC Bank with a strike of 4%. ABC Bank and the customer have created a "band" within which the customer will pay LIBOR plus the borrowing spread of 200 bps. If LIBOR drops

---

[7] Refer to the example for Cap with modifications, see Hull 518.

below the floor, the customer compensates ABC Bank. If LIBOR rises above the cap, ABC Bank compensates the borrower. The customer has foregone the benefit of reduced interest rates should LIBOR ever fall below 4%. In this example, the customer never pays more than 8% or less than 6%.

The customer is hedged against a rise in interest rates; however it enjoys limited benefits of falling rates. In a collar, the customer pays the index between the strike rates. However, if the index rises above the cap strike rate, ABC Bank compensates the customer the difference between the strike rate and the index. If the index rate falls below the floor strike, the customer pays ABC Bank the difference between the strike and the index rate.

4.4 Mathematical pricing

The price of a Cap is the sum of the price of the caplets that make up the Cap. Similarly, the value of a floor is the sum of the sequence of individual put options, often called floorlets that make up the floor.

$$Cap = \sum_{i=1}^{n} Caplet_i \ , \ Floor = \sum_{i=1}^{n} Floorlet_i \qquad (4.4.1)$$

where

$$V(Caplet) = \frac{Notional \times \frac{d}{Basis}}{(1 + f \times \frac{d}{Basis})} \times e^{-rT} \times \left[FN(d_1) - XN(d_2) \right] \qquad (4.4.2)$$

where d is the numbers of days in the forward rate period.

Basis is the day basis o number of days per year used in the market (i.e. 360 or 365)

$$V(Floorlet) = \frac{Notional \times \frac{d}{Basis}}{(1 + f \times \frac{d}{Basis})} \times e^{-rT} \times \left[XN(-d_2) - FN(-d_1) \right] \qquad (4.4.3)$$

where

$$d_1 = \frac{\ln(F / X) + (s^2 / 2)T}{s \sqrt{T}}$$

$$d_2 = d_1 - s \sqrt{T}$$

The Collar protects from the risk of rising and falling interest rates. The purchase of a collar comprises the simultaneous purchase of a cap and a sale of a Floor with identical maturities, national principals and reference rates. This is a common strategy for reducing the cost of the premium to insure against an adverse movement in short term interest rates. The premium obtained from the sale of the floor will in most cases only partially offset the cost of the cap. This cost can be reduced by raising the strike of the floor, when the premium of the floor exactly matches that of the cap this is known as a **Zero Cost Collar**. The sale of a collar works vice versa.

The buyer of a collar wants to hedge against rising interest rates and lowers his hedging costs through the sale of a floor. Therefore, he benefits from possible declines in interest rates only down to the floor.

The buyer of an interest rate collar purchases and interest rate cap while selling a floor indexed to the same interest rate. Borrowers with variable-rate loans buy collars to limit effective borrowing rates to a range of interest rates between some maximum, determined by the cap rate, and a minimum, which is fixed by the floor strike price; hence, the term "collar." Although buying a collar limits a borrower's ability to benefit from a significant decline in market interest rates, it has the advantage of being less expensive than buying a cap alone because the borrower earns premium income from the sale of the floor that offsets the cost of the cap. A zero-cost collar results when the premium earned by selling a floor exactly offsets the cap premium.

The amount of the payment due to or owed by a buyer of an interest rate collar is determined by the expression

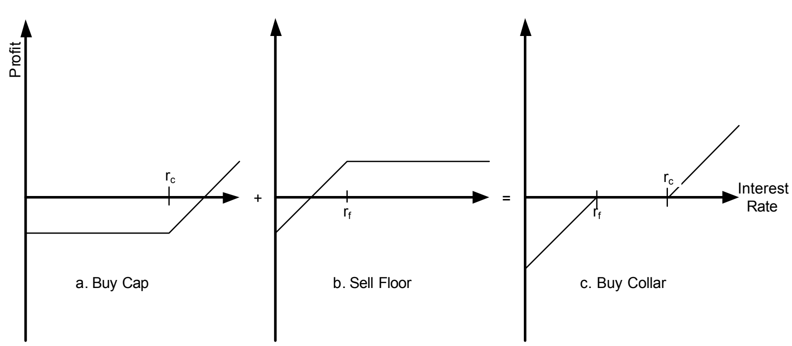
$$N \times \left[\max\left\{0, r - r_c\right\} - \max\left\{0, r_f - r\right\} \right] \times d_t \div 360 \qquad (4.4.4)$$

where, as before, N is the notional principal amount of the agreement, r_c is the cap rate, r_f is the floor rate, and d_t is the term of the index in days.

4.5 Theoretical Example

Figure 4.5.1 illustrates the payoff to buying a one-period zero-cost interest rate collar. If the index interest rate r is less than the floor rate r_f on the interest rate reset date, the floor is in-the-money and the collar buyer (who has sold a floor) must pay the collar counterparty an amount equal to $N \times (r_f - r) \times d_t \div 360$. When r is greater than r_f but less than the cap rate r_c, both the floor and the cap are out-of-the-money and no payments are exchanged. Finally, when the index is above the cap rate the cap is in-the-money and the buyer receives $N \times (r - r_c) \times d_t \div 360$.

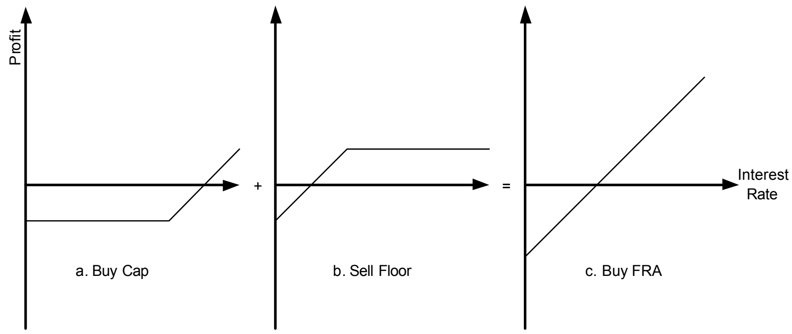
Figure 4.5.1 The Payoff to Buying a One-Period, Zero-Cost Collar[8]



[8] See Federal Reserve Bank of Richmond.

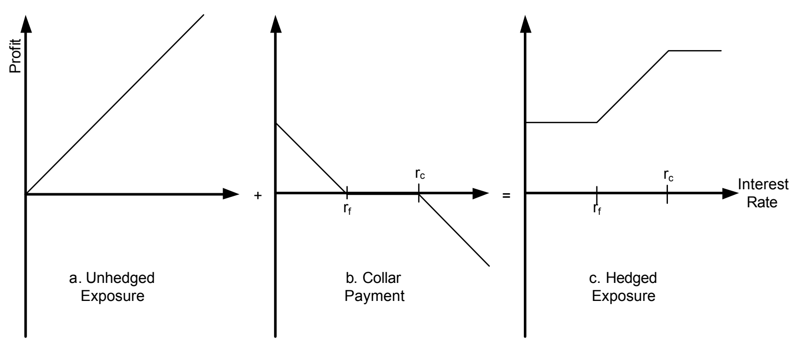
Figure 4.5.2 illustrates a special case of a zero-cost collar that results from the simultaneous purchasing of a one-period cap and sale of a one-period floor when the cap and floor rates are equal. In this case the combined transaction replicates the payoff of an FRA (Forward Rate Agreement) with a forward interest rate equal to the cap/floor rate. This result is a consequence of a property of option prices known as put-call parity.

Figure 4.5.2 Put-Call Parity[9]



More generally, the purchase of a cap and sale of a floor with the same notional principle, index rate, strike price, and reset dates produces the same payout stream as an interest rate swap with an All-In-Cost equal to the cap or floor rate. Since caps and floors can be viewed as a sequence of European call and put options on FRAs, buying a cap and selling a floor with the same strike price and interest rate reset and payment dates effectively creates a sequence of FRAs, all with the same forward rate. But note that an interest rate swap can be viewed as a sequence of FRAs, each with a forward rate equal to the All-In-Cost of the swap. Therefore, put-call parity implies that buying a cap and selling a floor with the same contract specifications results in the same payment stream that would be obtained by buying an interest rate swap.

Figure 4.5.3 The Effect of Buying an Interest Rate Collar on Interest Expense[10]



Finally, in this section, we will show the hedging uses of interest rate collars. Figure 4.5.3 illustrates the effect that buying a one-period, zero-cost collar has on the exposure to changes in market interest rates faced by a firm with outstanding variable-rate debt. The first panel depicts the firm's inherent or unhedged interest exposure, while the second panel illustrates the effect that buying a collar has on interest expense. Finally, the third panel combines the borrower's inherent exposure with the payoff to buying a collar to display the effect of a change in market interest rates on a hedged borrower's interest expense. Note that changes in

[9] See Federal Reserve Bank of Richmond.
[10] See Federal Reserve Bank of Richmond.

market interest rates can only affect the hedged borrower's interest expense when the index rate varies between the floor and cap rates. Outside this range, the borrower's interest expense is completely hedged.

4.6 Advantages and disadvantages

Advantages

1. Collars provide you with protection against unfavorable interest rate movements above the Cap Rate while allowing you to participate in some interest rate decreases.
2. Collars can be structured so that there is no up-front premium payable. While you can also set your own Cap Rate and Floor Rate, a premium may be payable in these circumstances.
3. The term of a Collar is flexible and does not have to match the term of the underlying bill facility. A Collar may be used as a form of short-term interest rate protection in times of uncertainty.
4. Collars can be cancelled (however there may be a cost in doing so – see the Early termination section for further details

Disadvantages

1. While a Collar provides you with some ability to participate in interest rate decreases, your interest rate cannot fall to less than the Floor Rate.
2. To provide a zero cost structure or a reasonable reduction in premium payable under the Cap, the Floor Rate may need to be set at a high level. This negates the potential to take advantage of favorable market rate movements.
3. You will be exposed to interest rate movements if the term of the Collar is shorter than that of the underlying bank bill facility.
4. There is no cooling off period.

Example

Company XYZ enters into a Zero Cost Collar with Bank ABC via the simultaneous purchase of a cap and a sale of a floor. The following details result from the deal being struck. Calculated on 90-day bank bill rate (BBSW) on the interest rate setting days.

Fixed Rate Payer	XYZ Pty Ltd
Fixed Rate Receiver	Bank ABC
Floating Rate Receiver	XYZ Pty Ltd
Floating Rate Payer	Bank ABC
Notional Principal	$1,000,000
Cap Rate	5.50%
Floor Rate	5.00%
Premium	nil
Floating Rate	Calculated on 90 day bank bill rate (BBSW) on the interest rate setting days
Interest Rate Setting Days	1 July, 1 October, 1 January, 1 April
Term	One year from 1 July 2001 to 1 July 2002

This means that every three months:

1. XYZ Pty Ltd will 'pay' Bank ABC the set 90-Day BBSW to a maximum of 5.50% per annum, but with a minimum of 5.00% on $1 million.
2. XYZ Pty Ltd will 'receive' from Bank ABC the set 90 day BBSW (i.e.: floating rate) on $1 million.

Bank ABC will net (1) and (2) and settle on the difference amount if BBSW exceeds 5.50% or is below 5.00%.

Let us look one year ahead. Assume the 90 day BBSW on the interest set days were:

1 July 2001	5.75
1 October 2001	5.25
1 January 2002	5.00
1 April 2002	4.75

Date From	Date To	Number of Days	*NP	Fixed Interest Rate		Floating Interest Rate (BBSW)		Difference between Fixed Interest Rate and Floating Interest Rate
			$ M	%	$	%	$	$
01/07/01	01/10/01	92	1	5.50	13,863.01	5.75	13,863.01	630.14
01/10/01	01/01/02	92	1	5.25	13,232.88	5.25	13,863.01	nil
01/01/02	01/04/02	90	1	5.00	12,328.77	5.00	12,328.77	nil
01/04/02	01/07/02	91	1	5.00	12,465.75	4.75	11,842.47	623.28

***NP = Notional Principal**

Notes: i. Interest is calculated using the simple interest formula and is paid in arrears.
ii. In the first and second quarters XYZ Pty Ltd receives a net amount from Bank ABC. There are no payments in the last two quarters.

5. Exotic caps and floors[11]

There is more contract trade on the international Over – The – Counter market with cash flows. They are similar to what we discussed in above sections, but deviate in one or more aspects. The deviations complicate the pricing methods considerably. Here we will give a few of those exotic securities.

A **Knock-out cap**: will at any time t_i give the standard payoff $C_{t_i}^{t}$ unless the floating rate $\ell(t+d,t)$ during the period $[ti-d,ti]$ has exceeded a certain level, so the payoff is zero.

Example

A Japanese exporter is very profitable when USD/YEN exceeds 115.00. However at the current spot of 100.00, cash flow is tight. They require interest rate protection. A normal 7% cap for 5 year costs 364 bps. Instead they buy a five year 7% cap which Knocks-Out when USD/YEN reaches 115.00. The total cost for this structure is 205 bps.

A **bounded cap**: is like an ordinary cap except that the cap owner will only receive the scheduled payoff if the payments received so far due to the contract does not exceed a certain pre-specified level. Therefore the ordinary cap payment, $C_{t_i}^{t}$, is to be multiplied with an indicator function. The payoff at the end of a given period will depend not only on the interest rate in the beginning of the period, but also on previous interest rates. Bounded cap is a path-dependent asset.[12]

[11] SeeCiberConta.
[12] See Jan Röman 26.

A **Flexible cap**: is an Interest Rate Cap where the buyer is only entitled to utilize the cap for a limited and pre-defined number of reset periods. The cap is automatically used if the underlying index, say LIBOR, is above the strike level. Once the number of "uses" equals the limit, the cap can no longer be used by the buyer. As a result of this limited protection, the premium is lower than a traditional cap. A traditional cap can be thought of as a Flexible Cap where the limited number of "uses" equals the maximum possible number of uses. The Flexible Cap therefore allows the buyer to target their rate protection more closely while reducing the premium. However, the Flexible Cap does expose the buyer to more risk than the traditional cap. The Flexible Cap is always automatically exercised. The automatic exercise feature of the Flexible Cap means that the cap will be used the first defined number of times that LIBOR is above the strike, even if only marginally above. A Flexible Cap where the buyer can choose when to exercise the cap is known as a Chooser Flexible Cap. A Flexible Cap where the notional amount increases each time the cap is not exercised is known as a Super Flexible Cap.

A **Flexible floor**: is an Interest Rate Floor where the buyer is only entitled to utilize the floor for a limited and pre-defined number of reset periods. The floor is automatically used if the underlying index, say LIBOR, is below the strike level. Once the number of "uses" equals the limit, the floor can no longer be used by the buyer. As a result of this limited protection, the premium is lower than a traditional floor. A traditional floor can be thought of as a Flexible Floor where the limited number of "uses" equals the maximum possible number of uses. The Flexible Floor therefore allows the buyer to target their rate protection more closely while reducing the premium. However, the Flexible Floor does expose the buyer to more risk than the traditional floor. The Flexible Floor is always automatically exercised. The automatic exercise feature of the Flexible Floor means that the floor will be used the first defined number of times that LIBOR is below the strike, even if only marginally below. A Flexible Floor where the buyer can choose when to exercise the floor is known as a Chooser Flexible Floor. A Flexible Floor where the notional amount increases each time the floor is not exercised is known as a Super Flexible Floor.

Other exotic caps and floors are:

- Spread
- Ratchet
- Sticky
- Momentum
- Dual strike cap
- Chooser

6. Conclusion

As a summary to this paper, interest rate derivatives, Caps, Floors and Collars have options characters. Among all these characters, credit risk exposure is the most interested. Sellers of caps and floors face no credit risk, since neither type of agreement requires the buyer to make any payments other than the initial premium. But cap and floor buyers face the risk of nonperformance on the part of the seller any time a cap or floor goes "in-the-money"--that is, any time the seller is required to make payments to the buyer. Since a collar involves a short position in a floor and a long position in a cap, it can expose both the buyer and seller to counterparty credit risk.

The credit risk exposure faced by the buyer of an interest rate cap can be compared to the risk exposure of a fixed-rate payer in an interest rate swap. In both cases, the buyers face the risk that the seller will default when interest rates rise. Similarly, the buyer of an interest rate floor faces a credit risk exposure analogous to that of a floating-rate payer, or seller, of an interest rate swap. The total credit risk exposure in each case is determined by the cost of buying a replacement cap or floor.

References

Hull, John C. *Options, Futures, and Other Derivatives*. 5th ed. USA: Pearson, 2003.

Jarrow, Robert and Turnbull, Stuart. *Derivative Securities*. 2nd ed. Cincinnati: South-Western, 2000.

Röman, Jan R. M. *Lecture notes in Analytical Finance II*. Västerås: Mälardalen University, 2004.

Instruments of the Money Market. Federal Reserve Bank of Richmond, Virginia. 1998. 6 December 2004 <http://www.rich.frb.org/pubs/instruments/ch16.html>.

Interest Rate Cap. CiberConta. 2004. 6 December 2004 <http://ciberconta.unizar.es/bolsa/cap.htm>.

Knock-Out Caps and Floors. CiberConta. 2004. 9 December 2004 <http://www.ciberconta.unizar.es/bolsa/ko-cf.htm>.

Lecture 23. NYU Stern. 2004. 6 December 2004 <http://pages.stern.nyu.edu/~jcarpen0/courses/b403333/23caph.pdf>.

Life-long learner. Royal Institute of Technology, Stockholm. 2004. 6 December 2004 < http://www.lifelong-learners.com/opt/SYL/s2node11.php3>.

Rate Caps and Floors. FinancialCAD. 2004. 6 December 2004 < http://www.fincad.com/support/developerFunc/mathref/RATECAP.htm>.

The mechanics of option-styled interest rate derivatives. Association of Corporate Treasurers of Southern Africa. 2004. 6 December 2004 <http://www.actsa.org.za/articles/pspangendec99.htm>.